

# DIFFRACTIVE FACTORIZATION - A SIMPLE FIELD THEORY MODEL FOR $F_2^{diff}(\beta x_{\mathbb{P}}, Q^2; x_{\mathbb{P}}, t)$

Arjun Berera

*Department of Physics, Pennsylvania State University, University Park, PA 16802, U.S.A.*

## Abstract

Operator definitions of diffractive parton distribution functions are given. A distinction is made between the special case of “Regge factorization” to the general case of “diffractive factorization” with explicit expressions for  $F_2^{diff}(\beta x_{\mathbb{P}}, Q^2; x_{\mathbb{P}}, t)$  in both cases. A calculation from a simple field theory model is presented in the style of “constituent counting rules” for the behavior of the diffractive parton distribution functions when  $\beta \rightarrow 1$ , which corresponds to when the detected parton carries almost all of the longitudinal momentum transferred from the scattered hadron. A comment is made about the consistency of the model with the observed flattening of  $n(\beta)$  as  $\beta \rightarrow 1$ , which recently was reported by the H1-collaboration from their preliminary 1994 data.

To appear in Proceedings of the International Workshop on Deep-Inelastic Scattering and Related Topics, Rome, Italy 1996, ed. G. D’ Agostini

In this talk I will discuss factorization in diffractive DIS. In the general phenomena of diffractive hard scattering, the initial proton in DIS or even both protons at hadron collider participate in a hard process involving a very large momentum transfer, but one or at hadron colliders one or both hadrons is diffractively scattered, emerging with a small transverse momentum and the loss of a rather small fraction of longitudinal momentum.

As shown by CFS [1], hard factorization breaks down at leading twist for pure hadronically initiated hard diffraction processes. This is discussed further in my talk on double pomeron exchange [2] presented at this conference. However, at HERA we can hypothesize factorization for diffractive DIS.

In the first stage we hypothesize that the diffractive structure function  $F_2^{\text{diff}}$  can be written in terms of a diffractive parton distribution :

$$\frac{dF_2^{\text{diff}}(\beta x_{\mathbb{P}}, Q^2; x_{\mathbb{P}}, t)}{dx_{\mathbb{P}} dt} = x_{\mathbb{P}} \sum_a \int_{\beta} d\beta' \frac{d f_{a/A}^{\text{diff}}(\beta' x_{\mathbb{P}}, \mu; x_{\mathbb{P}}, t)}{dx_{\mathbb{P}} dt} \hat{F}_{2,a}(\beta/\beta', Q^2; \mu), \quad (1)$$

where  $\hat{F}_2$  is the same function which is convoluted with the inclusive parton densities to compute  $F_2$  of inclusive DIS. If for simplicity, we ignore  $Z$  exchange, then  $\hat{F}_{2,a}(\beta/\beta', Q^2; \mu) = e_a^2 \delta(1 - \beta/\beta') + \mathcal{O}(\alpha_s)$ .

In the second stage, we hypothesize that the diffractive parton distribution function has a particular form:

$$\frac{d f_{a/A}^{\text{diff}}(\beta x_{\mathbb{P}}, \mu; x_{\mathbb{P}}, t)}{dx_{\mathbb{P}} dt} = \frac{1}{8\pi^2} |\beta_A(t)|^2 x_{\mathbb{P}}^{-2\alpha(t)} f_{a/\mathbb{P}}(\beta, t, \mu). \quad (2)$$

Here  $\beta_A(t)$  is the pomeron coupling to hadron A and  $\alpha(t)$  is the pomeron trajectory. The function  $f_{a/\mathbb{P}}(\beta, t, \mu)$  defined above is the “distribution of partons in the pomeron”. I distinguish the “diffractive factorization” of Eq. (1) from the “Regge factorization” of Eq. (2). The latter is a special case of the former. The Ingelman-Schlein model [3] is synonymous with “Regge factorization”. The structure function  $F_2^{\text{diff}}(\beta x_{\mathbb{P}}, Q^2; x_{\mathbb{P}}, t)$  for the IS-model is obtained by inserting Eq. (2) into (1). An inconsistency of data to the IS-model does not also imply an inconsistency to diffractive factorization.

I now give operator definitions of the diffractive parton distribution. The diffractive distribution of a quark of type  $j \in \{u, \bar{u}, d, \bar{d}, \dots\}$  in a hadron of type A in terms of field operators  $\tilde{\psi}(y^+, y^-, \mathbf{y})$  evaluated at  $y^+ = 0, \mathbf{y} = 0$  is:

$$\begin{aligned} \frac{d f_{a/A}^{\text{diff}}(\beta x_{\mathbb{P}}, \mu; x_{\mathbb{P}}, t)}{dx_{\mathbb{P}} dt} &= \frac{1}{64\pi^3} \frac{1}{2} \sum_{s_A} \int dy^- e^{-i\beta x_{\mathbb{P}} P_A^+ y^-} \\ &\sum_{X, s_{A'}} \langle P_A, s_A | \tilde{\psi}_j(0, y^-, \mathbf{0}) | P_{A'}, s_{A'}; X \rangle \gamma^+ \langle P_{A'}, s_{A'}; X | \tilde{\psi}_j(0) | P_A, s_A \rangle. \end{aligned} \quad (3)$$

We sum over the spin  $s_{A'}$  of the final state proton and over the states  $X$  of any other particles that may accompany it. Similarly, the diffractive distribution of gluons in a hadron is

$$\begin{aligned} \frac{d f_{a/A}^{\text{diff}}(\beta x_{\mathbb{P}}, \mu; x_{\mathbb{P}}, t)}{dx_{\mathbb{P}} dt} &= \frac{1}{32\pi^3 \beta x_{\mathbb{P}} P_A^+} \frac{1}{2} \sum_{s_A} \int dy^- e^{-i\beta x_{\mathbb{P}} P_A^+ y^-} \\ &\sum_{X, s_{A'}} \langle P_A, s_A | \tilde{F}_a(0, y^-, \mathbf{0})^{+\nu} | P_{A'}, s_{A'}; X \rangle \langle P_{A'}, s_{A'}; X | \tilde{F}_a(0)_{\nu}^+ | P_A, s_A \rangle. \end{aligned} \quad (4)$$

The proton state  $|P_A, s_A\rangle$  has spin  $s_A$  and momentum  $P_A^\mu = (P_A^+, M_A^2/[2P_A^+], \mathbf{0})$ . We average over the spin. Our states are normalized to  $\langle k|p\rangle = (2\pi)^3 2p^+ \delta(p^+ - k^+) \delta^2(\mathbf{p} - \mathbf{k})$ . The tilde on the fields  $\tilde{\psi}_j(0, y^-, \mathbf{0})$  and  $\tilde{F}_a(0, y^-, \mathbf{0})^{+\nu}$  is to imply that they are multiplied by an exponential of a line integral of the vector potential as shown in [4].

The diffractive parton distributions are ultraviolet divergent and require renormalization. It is convenient to perform the renormalization using the  $\overline{\text{MS}}$  prescription, as discussed in [5,6]. This introduces a renormalization scale  $\mu$  into the functions. In applications, one sets  $\mu$  to be the same order of magnitude as the hard scale of the physical process.

The renormalization involves ultraviolet divergent subgraphs. Subgraphs with more than two external parton legs carrying physical polarization do not have an overall divergence. Thus the divergent subgraphs are the same as for the ordinary parton distributions. We conclude that the renormalization group equation for the diffractive parton distributions is

$$\mu \frac{d}{d\mu} \frac{df_{a/A}^{\text{diff}}(\beta x_{\mathcal{P}}, \mu; x_{\mathcal{P}}, t)}{dx_{\mathcal{P}} dt} = \sum_b \int_{\beta x_{\mathcal{P}}}^1 \frac{dz}{z} P_{a/b}(\beta x_{\mathcal{P}}/z, \alpha_s(\mu)) \frac{df_{b/A}^{\text{diff}}(z, \mu; x_{\mathcal{P}}, t)}{dx_{\mathcal{P}} dt} \quad (5)$$

with the same DGLAP kernel [7],  $P_{a/b}(\beta x_{\mathcal{P}}/z, \alpha_s(\mu))$ , as one uses for the evolution of ordinary parton distribution functions.

The diffractive parton distribution  $d f_{a/A}^{\text{diff}}(\beta x_{\mathcal{P}}, \mu; x_{\mathcal{P}}, t)/dx_{\mathcal{P}} dt$ , like the ordinary parton distribution, is essentially not calculable using perturbative methods. Recall, however, that it is possible to derive “constituent counting rules” that give predictions for ordinary parton distributions  $f_{a/A}(x, \mu)$  in the limit  $x \rightarrow 1$  for not too large values of the scale parameter  $\mu$  in the sense of the analysis by Brodsky and Farrar [8]. In the same spirit, in [4] we have considered the diffractive parton distributions in the limit  $\beta \rightarrow 1$ .

In our model the “pomeron” is represented by a 2-gluon exchange. Inherently the pomeron involves soft physics and its dynamics are unknown from QCD. However 2-gluon models have been successful in describing some aspects of the hard physics in diffractive hard processes. Within the context of our model, we find in the limit  $\beta \rightarrow 1$ , that there is an exact separation between the hard partonic physics, which is measured, and the soft pomeron (or better stated colorless exchange) physics, which is required in order for the proton to diffractively scatter into the final state. In spacetime the interpretation is this kinematic limit forces the entire “pomeron” to be probed as a pointlike object.

We find that the diffractive gluon distribution behaves as  $(1 - \beta)^p$  for  $\beta \rightarrow 1$  at moderate values of the scale  $\mu$ , say 2 GeV, with  $0 \leq p \leq 1$ . The choice  $p \approx 0$  corresponds to an effectively massless final state gluon, while  $p \approx 1$  corresponds to an effective gluon mass. For the diffractive quark distribution we find they behave as  $(1 - \beta)^2$ . However, suppose that we interpret the calculation as saying that the diffractive distribution of gluons is proportional to  $(1 - \beta)^0$  for  $\beta$  near 1 when the scale  $\mu$  is not too large. Then the evolution equation for the diffractive parton distributions will give a quark distribution that behaves like

$$\frac{df_{q/A}^{\text{diff}}(\beta x_{\mathcal{P}}, \mu; x_{\mathcal{P}}, t)}{dx_{\mathcal{P}} dt} \propto (1 - \beta)^1, \quad (6)$$

when the scale  $\mu$  is large enough that some gluon to quark evolution has occurred, but not so large that effective power  $p$  in  $(1 - \beta)^p$  for the gluon distribution has evolved substantially

from  $p = 0$ . A signature of this phenomenon is that the diffractive quark distribution will be growing as  $\mu$  increases at large  $\beta$ , rather than shrinking. Perhaps this is seen in the data [9].

*Post Conference Comment :* Let us examine the  $\beta$ -dependence of the pomeron intercept, as reported by the H1-collaboration at this conference from their 1994 preliminary data [10]. To clarify conventions, the parameterization and notation used by H1 for the diffractive structure function is

$$F_2^D = A(\beta, Q^2) \frac{1}{x_{\mathbb{P}}^n}. \quad (7)$$

H1 is reporting that  $n$  depends on  $\beta$ , so more appropriately  $n(\beta)$ .

$\beta$  is a kinematic variable associated with the hard physics. I know of no theoretical argument that precludes  $\beta$  from affecting the soft physics for general value of  $\beta$ . However the hard/soft separation found in our model (discussed after eq. (5)) as  $\beta \rightarrow 1$  suggests that the  $\beta$  dependence in the soft physics should diminish in this limit. It is therefore reassuring to see from the 94 H1-preliminary data that the measured curve for  $n(\beta)$  flattens as  $\beta \rightarrow 1$ .

I clarify that a  $\beta$ -dependence in the intercept does not imply a breakdown of diffractive factorization. For this the intercept (or equivalently  $n$ ) needs a  $Q^2$ -dependence. which is not found in Zeus '93 and up to H1 '94 data [9,10].

## ACKNOWLEDGMENTS

I thank R. Ball and V. Del Duca for their invitation. This work was supported by the U.S. Department of Energy.

## REFERENCES

- [1] J.C. Collins, L. Frankfurt, and M. Strikman, *Phys. Lett. B* **307**, 161 (1993).
- [2] A. Berera, these proceedings.
- [3] G. Ingelman and P. Schlein, *Phys. Lett. B* **152**, 256 (1985).
- [4] A. Berera and D. E. Soper, "Behavior of Diffractive Parton Distribution Functions", hep-ph/9509239, (in press *Phys. Rev. D* 1996).
- [5] J. C. Collins and D. E. Soper, *Nucl. Phys. B* **194**, 445 (1982).
- [6] G. Curci, W. Furmanski and R. Petronzio, *Nucl. Phys. B* **175**, 27 (1980).
- [7] V. N. Gribov and L. N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972); Yu. L. Dokshitzer, *Sov. Phys. JEPT* **56**, 641 (1977); G. Altarelli and G. Parisi, *Nucl. Phys. B* **26**, 298 (1978).
- [8] S. J. Brodsky and G. Farrar, *Phys. Rev. D* **11**, 1309 (1975).
- [9] ZEUS Collaboration (M. Derrick, *et al.*), *Z. Phys. C* **70**, 391 (1996); H1 Collaboration, (T. Ahmed *et al.*), *Phys. Lett. B* **348**, 681 (1995).
- [10] P. Newman (H1 Collaboration), these proceedings.